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A TWO-SCALE MODEL OF STIFFNESS FOR 1D TENSILE CRACK OPENING, CLOSURE AND HEALING

Chloé Arson (chloe.arson@ce.gatech.edu)
School of Civil and Environmental Engineering, Georgia Institute of Technology

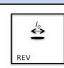
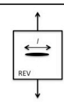
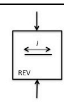
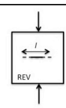
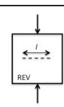
Problem Statement: Modeling Challenges

- No consensus on the definition of healing: effect on deformation, permeability, stiffness?
- Different theoretical frameworks for: crack opening (LEFM/CDM), closure (CDM), healing (DMT)
- No justification of the external and internal variables employed in coupled models (Arson et al. [4]).

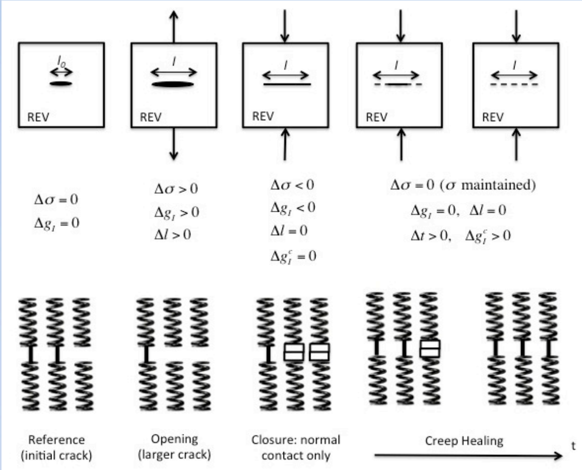
Atomic Scale – Molecular Dynamics, Electronic Forces
Crack Scale – LEFM, DMT models, Creep deformation
REV Scale – CDM, Effective Properties
Fracture Scale – LEFM, Interface energy

Crack opening + healing: Wiederhorn & Townsend [2], Wool [3]
Crack opening + closure: Chaboche [6]
Crack opening + healing: Senseny et al. [1]
Crack opening: Mazars & Pijaudier-Cabot [5]
Crack opening + healing: Ju et al. [7], Voyiadjis et al. [8]

Conceptual Stiffness Model

1. Reference State	$\tilde{\mathbf{C}}(l,g_I^c)=\mathbf{C}_{0,l=0}$	
2. Open Crack	$\tilde{\mathbf{C}}(l,g_I^c)=\mathbf{C}(l\|g_I^c)$	
3. Closed Crack	$\tilde{\mathbf{C}}(l,g_I^c)=\mathbf{C}(l\|g_I^c)-H(-g_I)\left[\mathbf{C}_0-\mathbf{C}(l\|g_I^c)\right]$	
4. Healing Crack	$\tilde{\mathbf{C}}(l,g_I^c)=\mathbf{C}(l\|g_I^c)-H(-g_I)\left[\mathbf{C}_0-\mathbf{C}(l\|g_I^c)\right]+H(+g_I)\left[\mathbf{C}_0-\mathbf{C}^*(g_I^c\|l)\right]$	
5. Healed Crack	$\tilde{\mathbf{C}}(l,g_I^c)=\mathbf{C}_{0,l\neq 0}$	

Formulation: Thermodynamic Framework



- **First Law of Thermodynamics** $W_{ext}=U+\Pi+Q$
- **Inequality of Clausisus-Duhem** $\dot{W}_{ext}-\dot{\Pi}-\dot{\psi}_s\geq 0$

- **Free energy:** LEFM potential energy

$$g_I=-\frac{\partial\psi_s}{\partial l}=-\frac{\partial\Pi}{\partial l},\quad l^c=-\frac{\partial\psi_s}{\partial g_I^c}=-\frac{\partial\Pi}{\partial g_I^c}$$

- **Complementary evolution laws**
  - **Crack opening:** rate-dependent (CDM)

$$f_d\left(g_I,l,g_I^c\right)=g_I-g_I^R\left(l,g_I^c\right),\quad g_I^R\left(l,g_I^c\right)=\left[g_I(l)-g_I^c\right]$$

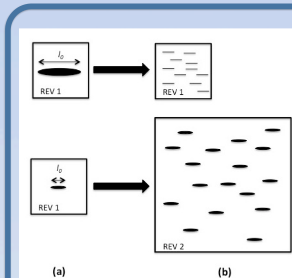
  - **Crack closure:** unilateral effects – stiffness model

$$\psi_s\left(\boldsymbol{\varepsilon}^e,l,g_I^c\right)=\frac{1}{2}\boldsymbol{\varepsilon}^e:\tilde{\mathbf{C}}\left(l,g_I^c\right):\boldsymbol{\varepsilon}^e$$

  - **Crack healing:** rate-dependent (DMT): $\frac{\partial g_I^c}{\partial t}=D\frac{\partial^2 g_I^c}{\partial z^2}$

Discussion, Perspectives, Broader Impacts

- Free energy (and stiffness) accounts for crack opening, closure and healing.
- Crack length *l* plays the role of a rate-independent damage variable. It is the longest actual length ever reached by the crack – not the actual crack length.
- Crack closure is not governed by any complementary evolution law other than the damage criterion (unilateral effects).
- The critical energy release rate is both the healing internal variable and a damage hardening variable.
- By construction, the model does not resort to an intermediate variable commonly defined as incremental net damage (difference between damage and recovery occurring during the same load increment).



Proposed model: equivalent to a **diffuse damage model for identical parallel micro-cracks**.
Parallel **tensile cracks in civil engineering**: columns, flexural beams…
Use of **self-healing cement-based materials**: an alternative to member replacement…

References

1. Senseny, P.E., Hansen, F.D., Russell, J.E., Carter, N.L., Handin, J.W., Mechanical Behaviour of Rock Salt: Phenomenology and Micromechanisms, Int. J. Rock Mech. Min. Sci. & Geomech. Abstr., 29:4, 1992, 363-378
2. Wiederhorn, S.M., Townsend, P.R., Crack healing in Glass, Proceedings of the 72nd Annual Meeting of the American Ceramic Society, 1970, 486-489
3. Wool, R.P., Self-healing materials: a review, Soft Matter, 2008, 4, 400–418
4. Arson, C., Xu, H., Chester, F.M., On The Definition Of Damage In Time-Dependent Healing Models For Salt Rock, Géotechnique Letters, 2, 2012, 67-71
5. Mazars, J., Pijaudier-Cabot, G., From Damage to Fracture Mechanics and Conversely: A Combined Approach, International Journal for Solids and Structures, 33:20-22, 1996, 3327-3342
6. Chaboche, J.L., 1993. Development of Continuum Damage Mechanics for Elastic solids Sustaining Anisotropic and Unilateral Damage. International Journal of Damage Mechanics, 2:311-329
7. Ju, J.W., Yuan, K.Y., Kuo, A.W., Novel Strain Energy Based Coupled Elastoplastic Damage and Healing Models for Geomaterials - Part I: Formulations, International Journal of Damage Mechanics, 21, 2012, 525-549
8. Voyiadjis, G.Z., Shojaei, A., Li, G., A generalized coupled viscoplastic–viscodamage–viscohealing theory for glassy polymers, International Journal of Plasticity, 28, 2012, 21–45